Optical Clocks

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„Experimental Quantum Metrology“
Head of Group: Piet O. Schmidt

„Quantum Sensors with Cold Ions“
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“Sub-Hz lasers and high performance cavities”
Project Leader: Thomas Kessler

Task Groups:
„Variations of Fundamental Constants“, „Transportable Ultra-Stable Clocks“
Outline

- Definition and Measurement of Time
- Time and Frequency Metrology
- Ingredients of an Optical Clock - Today’s State of the Art
  - Natural Reference - Candidates
  - Atom/Ion Traps
  - Local Oscillator
  - Frequency Comb
- Applications and Future Developments
Definition and Measurement of Time

• **Greenwich Mean Time**
  - global standard since 1884
  - 1 s = 1 / 86400 of the mean solar day

• **Mechanical Clocks**
  \[ \Delta t \sim 1 \text{ s/d} \]
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- **Quartz Clocks** ($f_{\text{res}} \sim \text{MHz}$)
  1930-40s: $\Delta t \sim 1 \, \text{ms/d}$
Definition and Measurement of Time

- **Greenwich Mean Time**
  - global standard since 1884
  - $1 \text{s} = \frac{1}{86400}$ of the mean solar day

- **Ephemeris Time**
  - adopted by the CGPM in 1960
  - $1 \text{s} = \frac{1}{31,556,925.9747}$ of the tropical year of 0. January 1900 at 12$^h$ UT

- **Mechanical Clocks**
  - $\Delta t \sim 1 \text{s/d}$

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  - 1930-40s: $\Delta t \sim 1 \text{ms/d}$

- **Atomic Cs-Clocks** ($f_{\text{res}} = 9.2 \text{ GHz}$)
  - 1955 Essen's clock: $\Delta t \sim 10 \mu\text{s/d}$
  - today: $\Delta t < 1 \text{ ns/d}$
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• **TAI (Temps Atomique International)**
  - since 1967

   “Duration of 9 192 631 770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the $^{133}$Cs atom”

• **Mechanical Clocks**
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- **Optical Clocks** \((f_{\text{res}} \sim 1000 \text{ THz})\)
Clock Operation (optical)

- Trapped ion or atoms (long term reference)
- Local Oscillator
- Frequency Comb
- Laser \( f \sim 1000 \text{ THz} \)
- ULE cavity (Flywheel)
- 10 Hz linewidth
- 31.08.2010
Time & Frequency Metrology
Uncertainty of Quantities in today’s Metrology

from BIPM Summer School of Metrology 2007
Time and Frequency Metrology

Stability

Accuracy $\delta\nu/\nu_0$

Example:

Allan variance

Fractional frequency instability as a function of averaging time

Fractional frequency:

\[ y = \frac{f(t) - f_0}{f_0} \]

\[ \sigma_y^2(\tau) = \left< \frac{1}{2} (\bar{y}_{i-1} - \bar{y}_i)^2 \right> \]

where

\[ \bar{y}_i = \frac{1}{\tau} \int_{t_i}^{t_{i+\tau}} y(t) dt \]
Allan variance

Fractional frequency instability as a function of averaging time

Fractional frequency:

\[ y = \frac{f(t) - f_0}{f_0} \]

Quality factor:

\[ Q = \frac{\nu_0}{\Delta \nu} \]

\[ \sigma_y(\tau) = \frac{1}{K \cdot Q \cdot \frac{S}{N} \left( \frac{T_c}{\tau} \right)^{1/2}} \]
Fractional frequency instability as a function of averaging time

Fractional frequency:

\[ y = \frac{f(t) - f_0}{f_0} \]

Stability needed to measure \( \nu_0 \) with this precision

BUT:
Is it accurate?
Accuracy / Systematic Errors

Example
1e\- atom: \( S = 1/2 \)
nuclear spin: \( I = 1/2 \)

\[
\begin{align*}
{^2P_{3/2}} & \quad 
{^2P_{1/2}} \\
{^2S_{1/2}} & \quad \text{e.g. 121 nm in H (= THz)}
\end{align*}
\]

Sytematic Shifts can be due to:

- Magnetic Fields
- Electric Fields / Blackbody radiation
- Collisions
  - Doppler shifts
  - Lock servo errors
  ...

\[
\begin{align*}
F = 0 & \quad F = 1 \\
-1, 0, 1 & \quad = m_F
\end{align*}
\]

\[
\begin{align*}
F = 0 & \quad \sim \text{GHz} \\
F = 1 & \quad \text{magnetic sublevels}
\end{align*}
\]

spin - orbit coupling (fine-structure)
\( J = |L \pm S| \)

HF-structure
\( F = |I \pm J| \)
Improvement through Optical Clocks

Accuracy $\Delta \nu / \nu_0$

Systematic shifts proportional to $\nu_0$: 1st and 2nd order Doppler shift

Systematic shifts with absolute order of magnitude: 1st and 2nd order Zeemann shift, Stark shifts (blackbody, light shift, etc…)

Stability

Allan Standard Deviation:

$$\sigma = \frac{1}{\pi \cdot S / N} \frac{1}{Q} \sqrt{\frac{T_c}{\tau}}$$

with $Q = \frac{\nu_0}{\Delta \nu}$

quality factor of transition

$\nu_0 = 9.19 \times 10^9$ Hz $\rightarrow \nu_0 \sim 10^{15}$ Hz
Ingredients of an Optical Clock
(A) Suitable Atoms
Atomic Transitions

\[ \gamma(e \rightarrow g) = \frac{4(2\pi\nu_0)^3}{3c^3} \alpha \left| \langle e|d|g \rangle \right| \]

→ forbidden transitions

But how to detect...

Example $^{199}$Hg$^+$

Quantum Jumps (e$^-$ - shelving)
Atomic Transitions

1e⁻ Systems

$^{199}\text{Hg}^+$, $^{171}\text{Yb}^+$, etc...

2e⁻ Systems

$^{87}\text{Sr}$, $^{27}\text{Al}^+$, $^{115}\text{In}^+$...

Ion Clocks ↔ Neutral Atom Clocks

$^2\text{S}_{1/2}$

$^1\text{S}_0$

$^2\text{P}_{1/2}$

$^2\text{D}_{5/2}$

$^3\text{P}_0$

Doppler Cooling 194 nm

282 nm Clock Laser

Clock Laser 698 nm

2e⁻ Systems

$^1\text{S}_0$

$^2\text{P}_{1/2}$

$^2\text{D}_{5/2}$

$^3\text{P}_0$
<table>
<thead>
<tr>
<th>Group</th>
<th>Period</th>
<th>Element</th>
<th>Symbol</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>Hydrogen</td>
<td>H</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>Lithium</td>
<td>Li</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
<td>Sodium</td>
<td>Na</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
<td>Magnesium</td>
<td>Mg</td>
</tr>
<tr>
<td>5</td>
<td>6</td>
<td>Barium</td>
<td>Ba</td>
</tr>
<tr>
<td>6</td>
<td>7</td>
<td>Francium</td>
<td>Ra</td>
</tr>
</tbody>
</table>

**Clocks**
(B) Atom / Ion Traps
Laser Cooling of Atoms

Nobel price 1997

"for development of methods to cool and trap atoms with laser light"

Steven Chu  
Claude Cohen-Tannoudji  
William D. Phillips
**Laser Cooling of Atoms**

**Doppler cooling:**
red detuned laser beams on strong transition $\gamma \sim 1 - 80$ MHz

- Laser cooling:
  - red detuned laser beams on strong transition $\gamma \sim 1 - 80$ MHz

- Evaporation

**Temperature scale of laser cooling**

- $1000$ K: atomic beam $v \sim 700$ m/s
- $300$ K: room temperature $v \sim$ some $100$ m/s
- $TDopp \sim 1$ mK $v \sim 1$ m/s
- sub-Doppler-cooling: $v \sim 1$ cm/s
- Evaporation

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31.08.2010

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Optical Dipole Traps for Neutral Atoms

Laser light distorts atomic energy levels

loading of Sr atoms into 1D lattice

Trap Depth ~ 50 µK
The Optical Lattice Clock

“magic wavelength”(*)

ground and excited state experience same Stark shift

The Optical Lattice Clock

Stark-Shift of $^1\text{S}_0$ and $^3\text{P}_{1,0}$

“magic wavelength”(*)

ground and excited state experience same Stark shift

Ion Traps

Paul trap

endcap trap

single Yb$^+$-ion
Ion Traps

Charged ions interact strongly with environment → trap with electric fields?

BUT: Non-Trapping Theorem

\[ \nabla \phi = \nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_0} = 0 \]

Trap Depth \( \sim 10^4 \) K

\[ \Psi = \frac{e^2 |E|^2}{4m\Omega^2} \]

Ponderomotive Potential

credit: W. Lange
Ion versus Neutral Atom Clocks

Optical Lattice Clocks with Neutrals

- $10^4 - 10^5$ atoms: high S/N
- short servo times
- high stability
- collisions
- higher order effects in lattice
- polarization dependence of lattice

Single Ion Optical Clocks

- ions are trapped in minimum of EM-field
- high level control of single ion
- no collisions
- storage times up to days/months
- limited short term stability
- higher need for ultra-stable laser
Today’s State of the Art

Comparison of 2 ion clocks ($^{27}\text{Al}^+/^{27}\text{Al}^+$):

- inaccuracy = $7.0 \times 10^{-18}$
- stability $\sigma \sim 2.0 \times 10^{-15}$ in 1s

Chou et al., PRL 104, 070802 (2010)

NIST and now at PTB (see P. Schmidt’s talk)

Best resolved atomic resonance ($^{87}\text{Sr}$):

Sr/Ca comparison (JILA/NIST):

- inaccuracy = $1.5 \times 10^{-16}$
- stability $\sim 3 \times 10^{-15}$ in 1s

Paris, Boulder, Tokyo, Braunschweig, London, Florence, Moscow, …

(C) Local Oscillator
Ultra-low expansion (ULE) glass:
- Coefficient of thermal expansion < 20 ppb/K
- Zero crossing close to room temperature

Spacer: 10 cm length

Optically contacted mirrors with Finesse $F > 100,000$

Linewidth: $\Delta f = \text{FSR} / F \sim 15$ kHz
Ultra-low expansion (ULE) glass:
- Coefficient of thermal expansion < 20 ppb/K
- Zero crossing close to room temperature

Spacer: 10 cm length

Optically contacted mirrors
with Finesse $F > 100,000$

1 Hz laser width:
$\sigma_v/\nu_0 = 3 \times 10^{-15}$
$\sigma_L = 3 \times 10^{-16} m$
Cavity Designs

**PTB horizontally mounted cavity**
Nazarova et al.
„Vibration-insensitive reference cavity for an ultra-narrow-linewidth laser“

**JILA vertical cavity**
Notcutt et al.
„Compact, thermal-noise-limited optical cavity for diode laser stabilization at $1 \times 10^{-15}$“
Optics Lett. 32, 641 (2007)

**NPL cut-out cavity**
Webster et al.
„Vibration insensitive optical cavity“
PRA 75, 011801 (R) (2007)

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Reduction of thermal noise in high performance optical reference cavities

Silicon as resonator material

Alternative mirror concepts

<table>
<thead>
<tr>
<th>Material</th>
<th>Q</th>
<th>$T_0$ (K)</th>
<th>$d\alpha/dT$ (K$^{-2}$)</th>
<th>Opt. transp</th>
</tr>
</thead>
<tbody>
<tr>
<td>Si</td>
<td>5x10$^7$</td>
<td>120</td>
<td>2.3 10$^{-8}$</td>
<td>IR</td>
</tr>
<tr>
<td>Si</td>
<td>17</td>
<td></td>
<td>1.2 10$^{-9}$</td>
<td></td>
</tr>
<tr>
<td>ULE</td>
<td>6x10$^4$</td>
<td>293</td>
<td>2.4 10$^{-9}$</td>
<td>VIS</td>
</tr>
</tbody>
</table>
Sub-Hz Lasers?

Reduction of thermal noise in high performance optical reference cavities

Silicon as resonator material

Alternative mirror concepts

![Diagram showing thermal noise and other noise sources over time constant (τ)]
Experimental Setup
(D) Frequency Combs
Nobel price 2005

"for their contributions to the development of laser-based precision spectroscopy, including the optical frequency comb technique"

John L. Hall

Theodor W. Hänsch
Optical Frequency Combs

- Mode locked Ti:Sapphire or Er-Fiber Lasers
- pulse duration 1 - 10 fs → \( f_{\text{rep}} = 100 \text{ MHz} - 1 \text{ GHz} \)
Optical Frequency Combs

\[ f_{\text{probe}} = m f_{\text{rep}} \pm f_0 \pm f_{\text{beat}} \]

- \( m \) measured with wavemeter
- \( f_{\text{rep}} \) locked to Cs-fountain
- \( f_0 \) by beating ground & 2nd harmonic
Required accuracy and stability for optical clock comparison has been demonstrated!

Ma et al., IEEE J. Quant. Electron. 43, 139 (2007)
Optical Frequency Combs
Applications and Future Developments
Clocks and Navigation

- Optical Clock in Ground Stations: Deep space missions
- Master Clock in Space: GNSS
  - improved location resolution
  - integrity of space segment, smaller prediction errors

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Satellite time</th>
<th>System time</th>
<th>Theoretical error over 2 hours / ns</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>Rubidium clock</td>
<td>Optical clock</td>
<td>0.59</td>
</tr>
<tr>
<td>II</td>
<td>Passive maser</td>
<td>Optical clock</td>
<td>0.11</td>
</tr>
<tr>
<td>III</td>
<td>Optical clock</td>
<td>Active maser</td>
<td>0.014</td>
</tr>
<tr>
<td>IV</td>
<td>Optical clock</td>
<td>Optical clock</td>
<td>0.002</td>
</tr>
</tbody>
</table>

Simulations by Institute of Communications & Navigation (DLR)

4 Swingbys near Venus, Jupiter, Earth at 300 km distance

Required accuracy: ±25 km
Equivalence Principle: fundamental constants need to be constant in time

\[ \frac{\Delta \alpha}{\alpha} = \left( -5.4 \pm 1.2 \right) \times 10^{-6} \]

\[ \frac{\Delta \mu}{\mu} = \left( 2.6 \pm 0.6 \right) \times 10^{-5} \]

Reinhold et al., PRL 96, 151101 (2006)
Laboratory Tests

Present status:

\[
\frac{\partial \ln \alpha}{\partial t} = (-2.4 \pm 2.7) \cdot 10^{-17} \text{ yr}^{-1}
\]

\[
\frac{\partial \ln R_y}{\partial t} = (0.0 \pm 3.2) \cdot 10^{-16} \text{ yr}^{-1}
\]

Dzuba et al. PRL 82 (1999)

Al+/Hg+: T. Rosenband et al., Science 319, 1808 (2008)
Shift of clock transition:  
\[ Z \equiv \frac{\Delta \nu}{\nu} = (1 + \varepsilon) \frac{\Delta U}{c^2} \]

non-zero if local position invariance doesn’t hold

\( \varepsilon \) tested to be < \( 7 \times 10^{-5} \)

Einstein Gravity Explorer (EGE) could provide test at the \( 2 \times 10^{-8} \) level

Schiller, Tino, Gill, Salomon et al. (2007)
Geodesy at $10^{-18}$

- Direct measurement of earth’s geopotential
- Current uncertainty: 30 - 50 cm

→ resolve height difference of $1$ cm

- Tracking tectonic plate movement 2-20 cm/yr

→ $\Delta v \sim 1$ cm/yr
Future Challenges

- Reliability (compact, portable clocks)
- Non destructive detection in optical lattice
- Can we use more than one ion? – new ion trap designs
- Entangled atoms / ions?
- Quantum logic clocks

\[ \Delta v_{Quad} \propto \nabla E \cdot \Theta = 0 \]
Scalable Chip Traps

- Laser cutting with pulsed 10ns tripled YAG at 355nm
- Low RF-loss AlN / sapphire wafer
- Au, Nb, Mo metalization
First Prototype (Rogers 4350B)

laser cut electrodes

stack of glued PC boards with laser cut electrodes

low pass 110 Hz

onboard RF-filtering with soldered SMD parts, connection of DC-electrodes via bonding of 30µm Au wire

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Ions in Chip Trap

Coulomb crystals of ytterbium-172 ions:
Ions in Chip Trap
Optical Clock Groups at PTB: